

A two-phase model of blood flow in a stenosed artery under the influence of external magnetic field

¹G.NEERAJA, ¹DINESH P.A and ²BABU. N

Abstract— A two-phase macroscopic model of blood in a stenosed catheterized artery under the influence of external transverse magnetic field is studied. The governing partial differential equations of the physical problem are solved analytically. The important blood flow characteristics in arteries namely resistive impedance and wall shear stress are discussed for various governing parameters like Hematocrit C, Hartmann number M and catheter size K involved in the model. It is observed that the resistive impedance and shear stress increases with increase in the stenosis height and decreases drastically with increase in the magnetic field parameter. The results are found in good agreement with the physiological conditions.

Index Terms— Two-phase, Macroscopic model, Resistive impedance, Magnetic field, Hartmann number, catheterized artery, blood flow.

1 INTRODUCTION

ONE of the major causes responsible for the circulatory disorders is the stenosis or atherosclerosis, which is the abnormal and unnatural growth in the arterial wall thickness. Hence, the mathematical modeling of this type of flows may help in proper understanding and prevention of arterial diseases. The arterial stenosis refers to the narrowing of an artery due to the development of atherosclerotic plaques rich in fatty substances like cholesterol along the arterial wall. Subsequent thickening and eventual hardening of the vessel wall leads to atherosclerosis – a cardiovascular disease which is one of the main causes of heart attack and various peripheral vascular diseases like cerebral accident (stroke) and paralysis (see Mitchell and Schwartz[1]). Since, the early investigation of Mann et al [2] several theoretical and experimental attempts have been made to study the blood flow characteristics due to the presence of a stenosis in the lumen of a blood vessel. The actual cause of stenosis is not well known but it has been suggested that the deposits of cholesterol on the arterial wall and proliferation of connective tissues may be responsible for the same (see Chaturani and Samy[3], Misra and Chakravorty [4], Shukla et al [5]). This can cause circulatory disorders by reducing the blood supply which may result in serious consequences like cerebral strokes and myocardial infarction.

The most characteristic biomagnetic fluid is the blood because the red blood cells (RBCs) contain the hemoglobin molecule, a form of iron oxides, which is present at uniquely high concentration in the mature red blood cells. Therefore, in an external magnetic field, the magnetic moment on the RBC varies due to blood oxygenation. It is found that the erythrocytes orient with their disc plane parallel to the magnetic field. The red blood cell is a major biomagnetic substance and the blood flow may be influenced by the magnetic field. The principles of magnetohydrodynamics have applications in medicine and biology and is of growing interest in the current literature of biomechanics (see Skalak [6], Sobin et al [7]). The subject of magnetohydrodynamics is the union of two widely separated fields namely electrodynamics and fluid dynamics, i.e., the motion of an electrically conducting fluid in the presence of electromagnetic field. The blood flow is influenced by electromagnetic field. Barnothy [8], reported that in general, the biological systems are affected by the application of an external magnetic field. Rao and Deshikachar [9], have given an excellent review of a good number of works concerning the effect of a magnetic field on the flow characteristics of blood through non-constricted single tube. Sud and Sekhon [10] have used the finite element method to analyze the effect of a magnetic field on blood flow through the human arterial system. Magnetic stress is caused by physical interactions between the magnetic field and biological substances (see Higashi et al [11]). As the red blood cells is a major biomagnetic substance (see Pauling and Coryell[12]), the blood flow may be influenced by the magnetic field.

With the evolution of the medical technology, catheters play a pivotal role in the modern medicine. Catheterization refers to a procedure in which a long, thin, flexible plastic tube (catheter) is inserted into an artery. The insertion of a catheter in an artery will naturally form an annular region

¹Department of Mathematics, M. S. Ramaiah Institute of Technology, Bangalore – 560 054

²Department of Mathematics, East West Institute of Technology, Bangalore – 560 091.

Email: n55raja@yahoo.co.in, dineshdpa@msrit.edu

between the walls of the catheter and artery. As a result, this will alter the flow field, like modifying the pressure distribution and increasing the resistance. Thus, it is very significant to study the flow of blood in a catheterized artery.

However, in view of the fact that blood is a suspension in reality, a two-phase model appears to be more appropriate. The increased impedance or the frictional resistance to flow and the wall shear stress will alter the velocity distribution when a catheter is inserted into an artery. A review of most of the experimental and theoretical investigations on artery catheterization has been presented by Srivastava and Srivastava [13]. Dash et al [14] analyzed the changed flow pattern in narrow artery when a catheter inserted into it and estimated the increase in the frictional resistance in the artery due to catheterization using the Casson fluid model for both steady and pulsatile flow. The changed flow patterns of pulsatile blood flow in a catheterized artery were studied by Sarkar and Jayaraman [15]. Shankar and Hemalatha [16] studied the flow of Herschel-Bulkley fluid in a catheterized artery and estimated the increase in the resistance to flow. Srivastava and Rashmi Srivastava [17] studied the particulate suspension blood flow through a narrow catheterized artery. A survey of the literature on the topic indicates that no effort has been made to observe the effects of an inserted catheter on flow characteristics using the two-phase macroscopic model of blood under the influence of external magnetic field through a stenosed artery. Hence, the main objective of the present work is to study a two-phase model of blood flow under the influence of external magnetic field in the case of stenosis catheterized artery which will be discussed in the following sections.

2 FORMULATION OF THE PROBLEM

The following assumptions were made in formulation of the physical problem:

- The stenosed artery is considered to be a cylindrical tube.
- The flow is fully developed and laminar and taken along the axis of \bar{z} .
- The blood flow is considered to be steady.
- Blood is considered to be Newtonian fluid model.
- Two-phase macroscopic model, that is, a suspension of red cells in plasma.
- The entrance effects and special effects have been ignored by considering the length of the blood vessel to be quite large compared to its diameter.
- The flow in the artery due to a constant pressure gradient along its axis.
- An of uniform transverse magnetic field (B_0) is applied on the catheterized artery.

- The flow of blood is within the annular region between artery of radius, R_0 and a catheter as a coaxial cylinder of radius K .
- L is the length of the artery

Flow geometry of the stenosed catheterized artery (see fig.1) is given by

$$R(z) = \begin{cases} R_0 - \frac{\delta_s}{2} \left(1 + \cos \frac{\pi z}{z_0}\right), & -z_0 \leq z \leq z_0 \\ R_0, & \text{otherwise} \end{cases} \quad (1)$$

where $2z_0$ is the length of stenosis and δ is the maximum height of the stenosis.

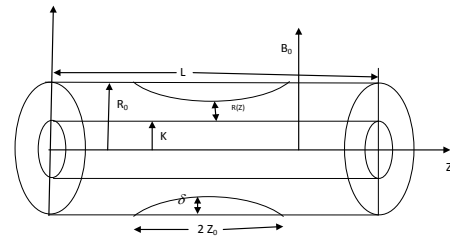


Fig. 1: Physical Configuration

The equations governing the conservation of mass and the linear momentum for the steady flow of a particle-fluid system, using a continuum approach, are expressed (see Srivastava and Srivastava [17] Drew [19]) as

For the fluid (Plasma phase)

$$(1-C) \left(u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right) = - (1-C) \frac{\partial p}{\partial z} + CS(u_p - u_f) + (1-C) \mu_s(C) \nabla^2 u_f - \sigma B_0^2 u_f, \quad (2)$$

$$(1-C) \rho_f \left(u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right) = - (1-C) \frac{\partial p}{\partial r} + (1-C) \mu_s(C) \left(\nabla^2 - \frac{1}{r^2} \right) v_f + CS(v_p - v_f), \quad (3)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r(1-C)v_f \right) + \frac{\partial}{\partial z} \left((1-C)u_f \right) = 0. \quad (4)$$

For the particle (Erythrocyte phase)

$$C \rho_p \left(u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right) = -C \frac{\partial p}{\partial z} + CS(u_f - u_p), \quad (5)$$

$$C \rho_p \left(u_p \frac{\partial v_p}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right) = -C \frac{\partial p}{\partial r} + CS(v_f - v_p), \quad (6)$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left(r C v_p \right) + \frac{\partial}{\partial z} \left(C u_p \right) = 0, \quad (7)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$ is the Laplacian operator; (r,z) is

cylindrical polar coordinates with z measured along the tube axis and r perpendicular to the axis of the tube; (u_f, u_p) and (v_f, v_p) are the velocity components of (fluid particle) phases along z and r directions, respectively; ρ_f and ρ_p are the actual densities of the material constituting the fluid and the particle phases, respectively; $(1-C)\rho_f$ and $C\rho_p$ are the fluid and particle phases density, respectively; C denotes the volume fraction density of the particles $\mu_s(C) = \mu_s$ is the suspension viscosity (apparent or effective viscosity), p is the pressure and S is the drag coefficient of interaction for the force exerted by one phase on the other and the subscripts f and p denote the quantities associated with the fluid and the particle phases, respectively.

The expression for drag coefficient S and viscosity of suspension μ_s have been chosen (see Srivastava and Srivastava [18]) as

$$\mu_s \cong \mu_s(C) = \frac{\mu_0}{1 - mc},$$

$$m = 0.07 \exp\left[2.49C + \left(\frac{1107}{T}\right) \exp(-1.69C)\right],$$

$$S = 4.5 \left(\frac{\mu_0}{a_0^2}\right) \frac{4 + 3 [8c - 3C^2]^{1/2} + 3C}{(2 - 3C)^2}, \quad (8)$$

where T is measured in absolute temperature ($^{\circ}K$), a_0 is the radius of a red cell and μ_0 is the plasma viscosity. The non-dimensional equations of flow are derived by following Sankar et al [20], we get

$$(1 - C) \frac{dp}{dz} = (1 - C) \frac{\mu_s}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) u_f + CS(u_p - u_f) - M^2 u_f, \quad (9)$$

$$C \frac{dp}{dz} = CS(u_f - u_p), \quad (10)$$

where $M^2 = \frac{\sigma B_0^2 R_0^2}{\mu_0}$ Hartmann number.

The corresponding boundary conditions are

$$u_f = 0 \quad \text{on} \quad r = R(z),$$

$$u_p = 0 \quad \text{on} \quad r = K. \quad (11)$$

Velocities for u_f and u_p are obtained by solving equations (9) and (10) with the boundary conditions from Equation(11), we get

$$u_f = \left[\frac{A_1}{A_3} K_0(Mr) - \frac{A_2}{A_3} I_0(Mr) - 1 \right] \frac{B}{M^2} \frac{dp}{dz},$$

$$u_p = \left[\frac{A_1}{A_3} K_0(Mr) - \frac{A_2}{A_3} I_0(Mr) - 1 \right] \frac{B}{M^2} \frac{dp}{dz} - \frac{1}{S} \frac{dp}{dz}$$

The for volumetric flow rate Q for the blood flow is

$$Q = 2\pi \int_K^R (r(1-c)u_f + rcu_p) dr,$$

$$Q = 2\pi (f_1(z) - f_2(z) - f_3(z) - f_4(z)) \frac{dp}{dz}.$$

The resistive impedance $\bar{\lambda}$ is physiological important hemodynamic indicator used in the study of resistance of blood flow in artery and it is given by

$$\bar{\lambda} = \frac{\Delta p}{Q} \quad \text{and} \quad \Delta p = \int_{-z_0}^{z_0} \frac{dp}{dz} dz,$$

where

$$A_1 = I_0(MK) - I_0(MR)$$

$$A_2 = K_0(MK) - K_0(MR)$$

$$A_3 = K_0(MR)I_0(MK) - K_0(MK) - I_0(MR)$$

$$B = \frac{1}{(1-c)\mu_r}$$

$$f_1(z) = \frac{A_1}{A_3} (MK K_1(MK) - MR K_1(MK))$$

$$f_2(z) = \frac{A_2}{A_3} (MR I_1(MR) - MK I_1(MK))$$

$$f_3(z) = \left[\frac{M^2}{2} (R^2 - K^2) \right] \frac{B}{M^2}$$

$$f_4(z) = \frac{c}{s} \frac{M^2}{2} (R^2 - K^2).$$

3 RESULTS AND DISCUSSION

The objective of the present mathematical model is to understand and bring out the effects of hematocrit C , the catheter size K and Hartmann number M on the resistive impedance and wall shear stress in a two layer magnetic fluid through a stenosed catheterized artery.

In order to discuss the results of the study quantitatively, computations are carried out to evaluate the analytical results obtained above numerically at the temperature of $37^{\circ}C$. The parameter values are selected as the particle diameter $2a_0$ (erythrocyte diameter) = $8\mu m$ and radius of the artery $R_0 = 70\mu m$ (see Srivastava and Rashmi Srivastava [17]).

The resistive impedance $\bar{\lambda}$ versus δ_s/R_0 has been shown in figures 2-4 for different values of Hartmann number M , Hematokrit C and catheter size K . In Figs. 2-4, the resistive impedance $\bar{\lambda}$ increases steadily with increase in the aspect ratio δ_s/R_0 . In Fig.2, the resistive impedance $\bar{\lambda}$ increases with increase in the magnetic strength M . The effect of external magnetic field strength M will reduces the blood flow in the artery in turn will resists the flow of blood. This will

leads to increase in the resistive impedance of the flow. These results are well satisfied with the physiological situation of the problem. Similarly, we observe that with an increase in the catheterized size K, the region of blood flow in the artery will reduces in turn the resistive impedance will increases. These results are depicted in Fig.3. An opposite behavior is observed in the case of various values of Hematocrit C, because of the volume of red blood cells penetrating in the flow region is increases will augments the resistive impedance of the flow is seen in Fig. 4.

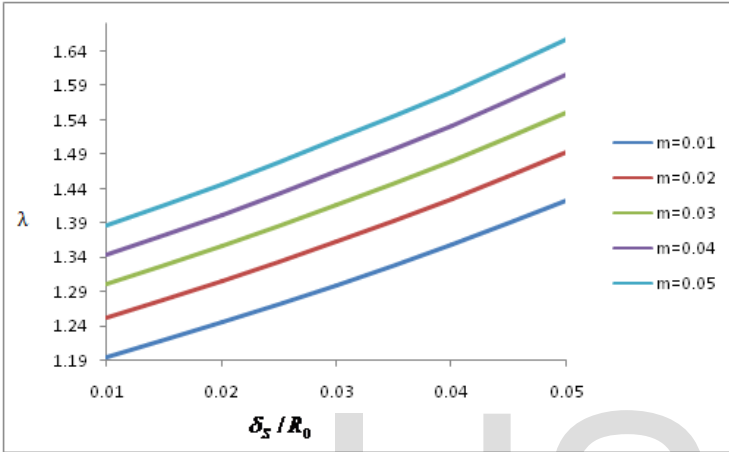


Fig. 2: Resistive impedance $\bar{\lambda}$ versus aspect ratio δ_s/R_0 for different values of M for C=0.2 K=0.1

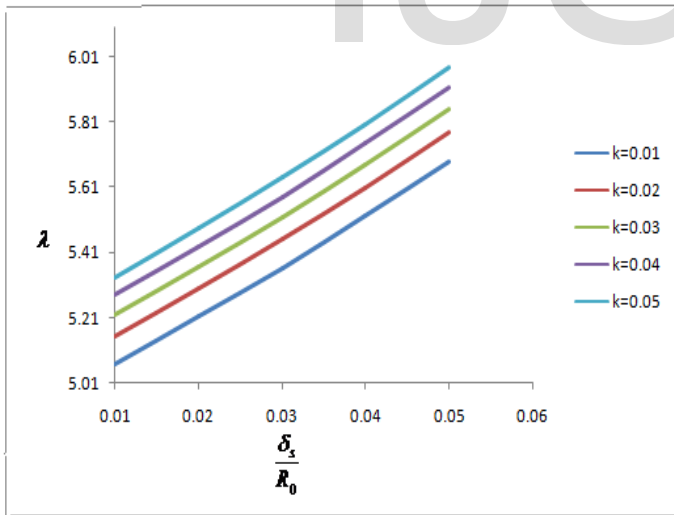


Fig. 3: Resistive impedance $\bar{\lambda}$ versus aspect ratio δ_s/R_0 for different values of K for C=0.02 M=10.

The shear stress $\tau = -\frac{R}{2} \frac{dp}{dz}$ is evaluated numerically

and the shear stress versus aspect ratio δ_s/R_0 has been shown in Tables 1-3, for different values of Hartmann number M, Hematocrit C and Catheter size K.

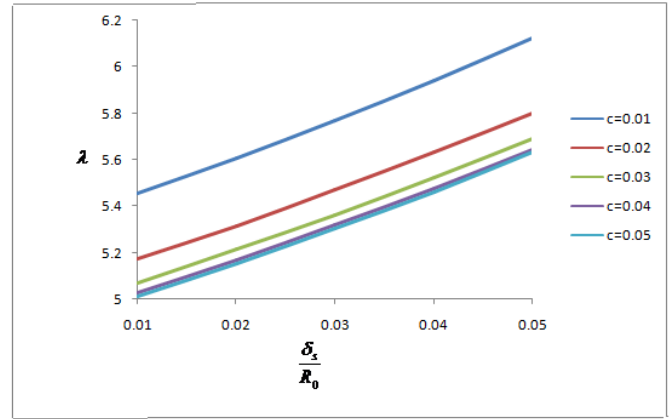


Fig. 4: Resistive impedance $\bar{\lambda}$ versus δ_s/R_0 for different values of C for M=10, K=0.02.

From the Table 1, it is observed that as magnetic parameter Hartmann number M increases the shear stress at the wall drastically decreases and it increases with increase in the aspect ratio δ_s/R_0 . In Table 2 and Table 3, it is observed that, the variations with K and C indicate the same behavior. But this decrease is very slow when compared to the variation of magnetic parameter.

Table 1 : Shear stress τ versus δ_s/R_0 for different values of m for C=0.2 K=0.1.

δ_s/R_0	M=10	M=20	M=30	M=40	M=50
0.1	0.0766	0.021	0.009	0.005	0.0035
0.2	0.0913	0.025	0.012	0.006	0.0041
0.3	0.113	0.031	0.014	0.008	0.0051
0.4	0.149	0.040	0.018	0.015	0.0066
0.5	0.221	0.058	0.026	0.016	0.0095

Table 2 : Shear stress τ versus δ_s/R_0 for different values of C for M=10 K=0.1.

δ_s/R_0	C=0.01	C=0.02	C=0.03	C=0.04	C=0.05
0.01	0.0498	0.0458	0.0452	0.0453	0.0458
0.02	0.0494	0.0464	0.0458	0.0459	0.0464
0.03	0.0501	0.0471	0.0464	0.0465	0.0470
0.04	0.0508	0.0478	0.0471	0.0472	0.0477

0.05	0.0515	0.0484	0.0477	0.0478	0.0484
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Table 3: Shear stress τ versus δ_s/R_0 for different values of K for M=10 C=0.02.

δ_s/R_0	K=0.1	K=0.2	K=0.3	K=0.4	K=0.5
0.01	0.0458	0.0459	0.0460	0.0461	0.0463
0.02	0.0465	0.0465	0.0467	0.0468	0.0476
0.03	0.0471	0.0472	0.0473	0.0475	0.0486
0.04	0.0475	0.0479	0.0479	0.0481	0.0493
0.05	0.0485	0.0485	0.0486	0.0489	0.0509

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